A modal bispectrum estimator for the CMB bispectrum

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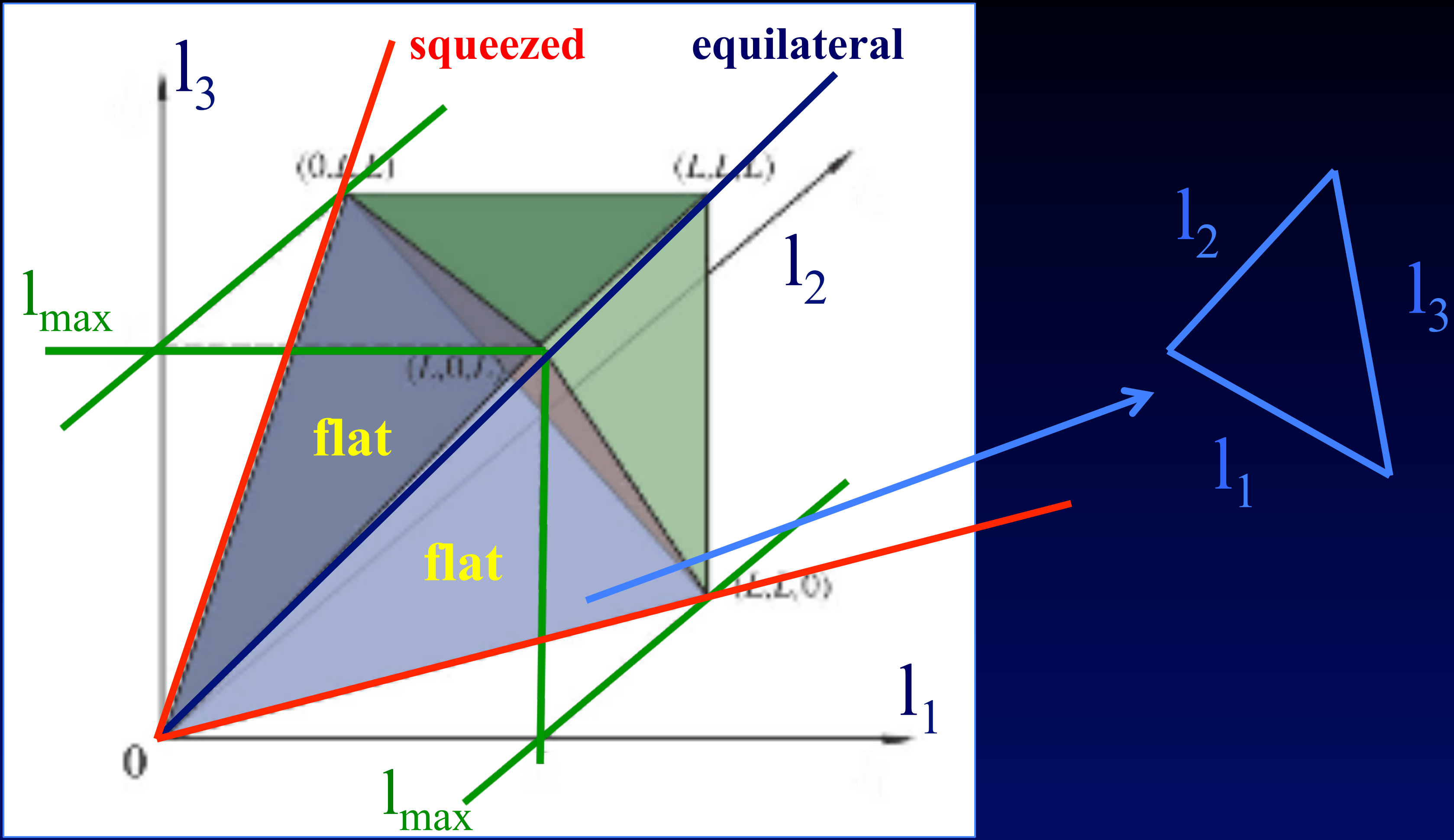
Fergusson, Liguori and Shellard (2010)

Outline

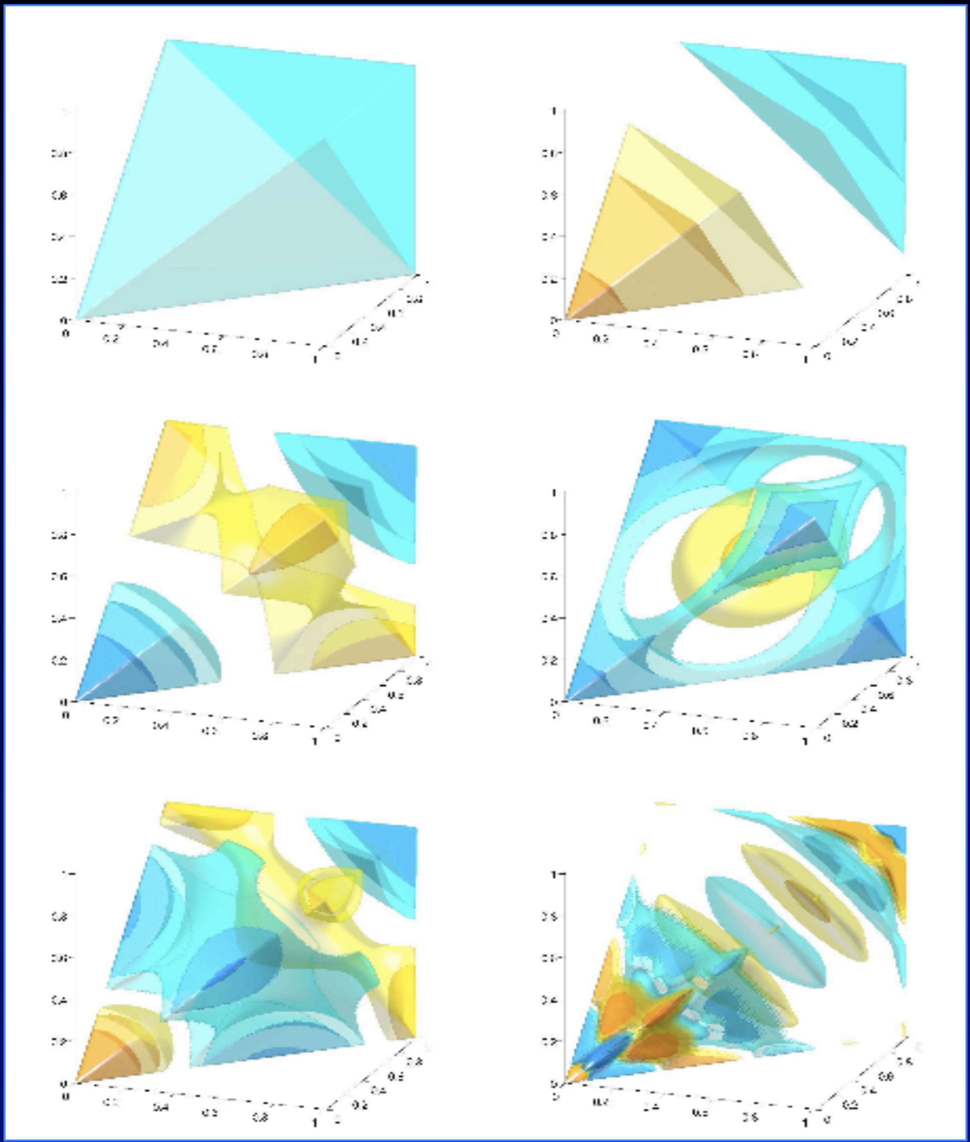
- Summary of the technique
  1. Polynomial modes
  2. What we measure:
     f_{NL}, mode spectrum, shape reconstruction
- Results from WMAP-5
  1. Model independent: mode spectrum and bispectrum reconstr.
  2. Scale independent shapes
  3. Running shape: feature in the inflaton potential
- Extension to *Planck*
- Summary

Bispectrum domain



$$B_{\ell_1\ell_2\ell_3} = \sum \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle a_{\ell_1}^{m_1} a_{\ell_2}^{m_2} a_{\ell_3}^{m_3} \right\rangle \; ; \; b_{\ell_1\ell_2\ell_3} = h_{\ell_1\ell_2\ell_3} B_{\ell_1\ell_2\ell_3}$$

Mode expansion

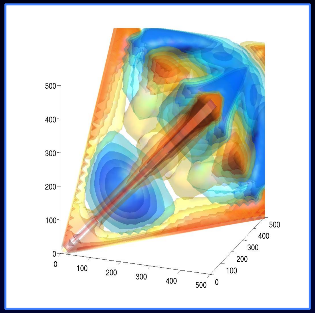


We define the scalar product:

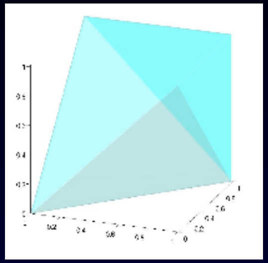
$$\langle f,g\rangle = \sum_{\ell_1\ell_2\ell_3} w_3(\ell_1,\ell_2,\ell_3) f(\ell_1,\ell_2,\ell_3) g(\ell_1,\ell_2,\ell_3)$$

We expand the bispectrum in terms of separable orthonormal functions defined in the shaded domain (tetrapyd) with scalar product above.

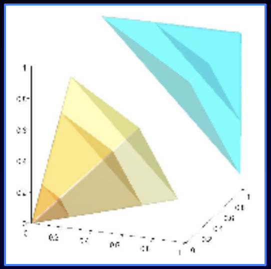
Mode estimation



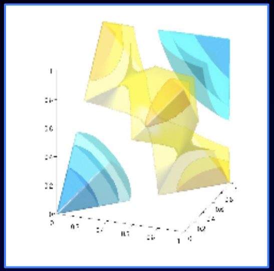
$= \beta_0$



$+ \beta_1$



$+ \beta_2$



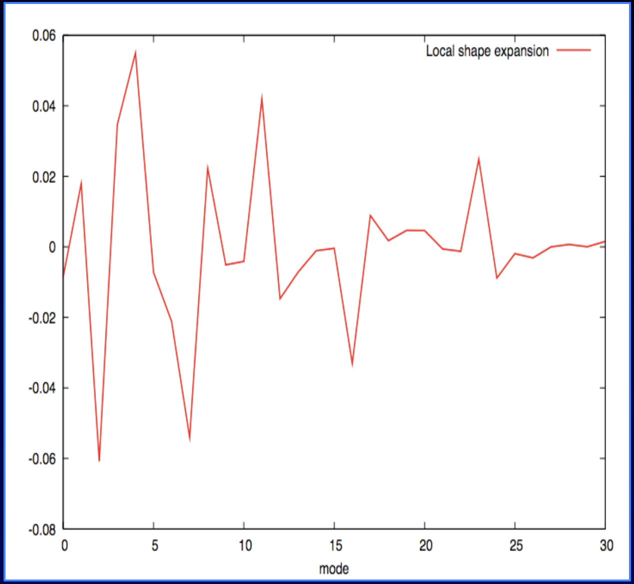
$+ \ldots$

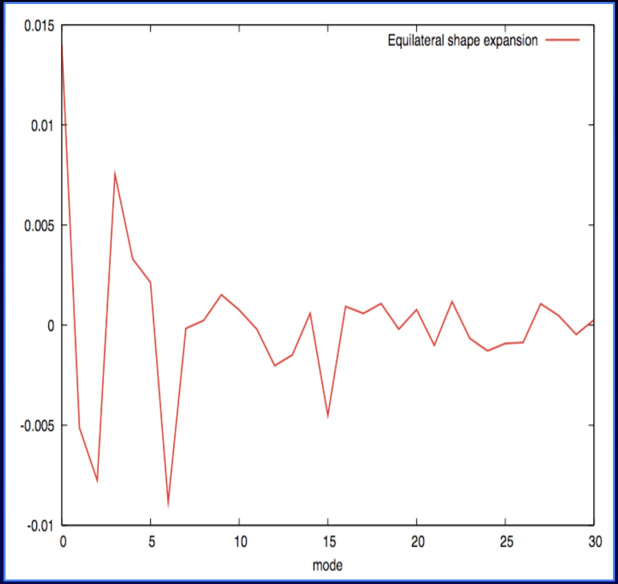
Goal: for a given dataset, extract best-fit β_i, $i=1,\ldots,p$

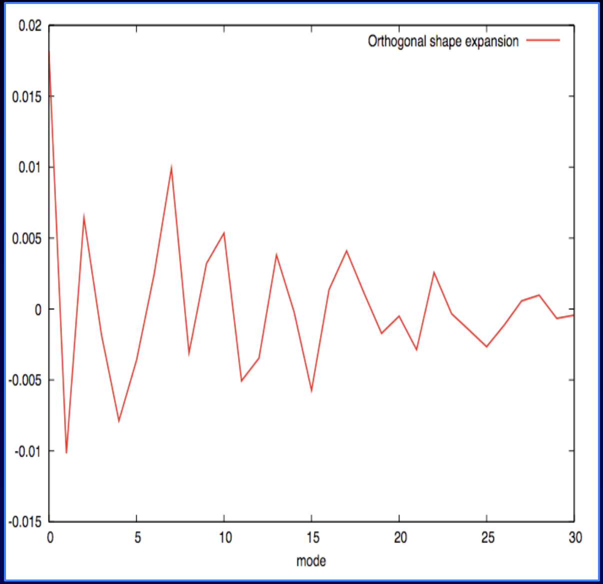
- The basis elements pictured on the right *are by construction factorizable*
- Apply position space cubic statistics by Komatsu, Spergel and Wandelt (2003) to each separable template on the right to estimate the amplitudes β_i
- Orthonormal basis ➔ β_i uncorrelated (in first approx.)

f_{NL} estimation

- Expand theoretical shape until a good level of correlation is achieved







- Extract mode amplitude from the data up to the highest mode in the shapes under study

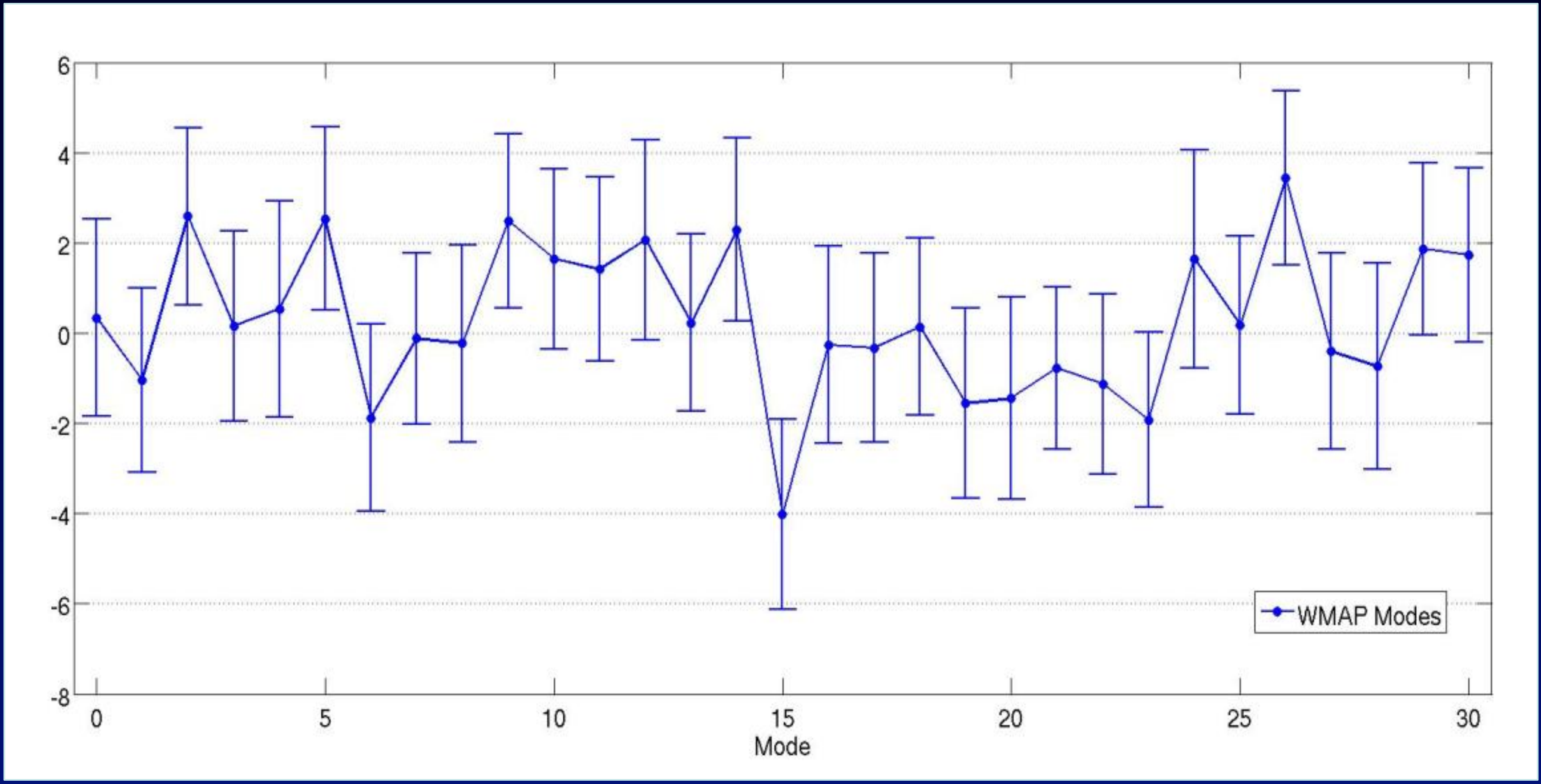
- Correlate to get f_{NL}

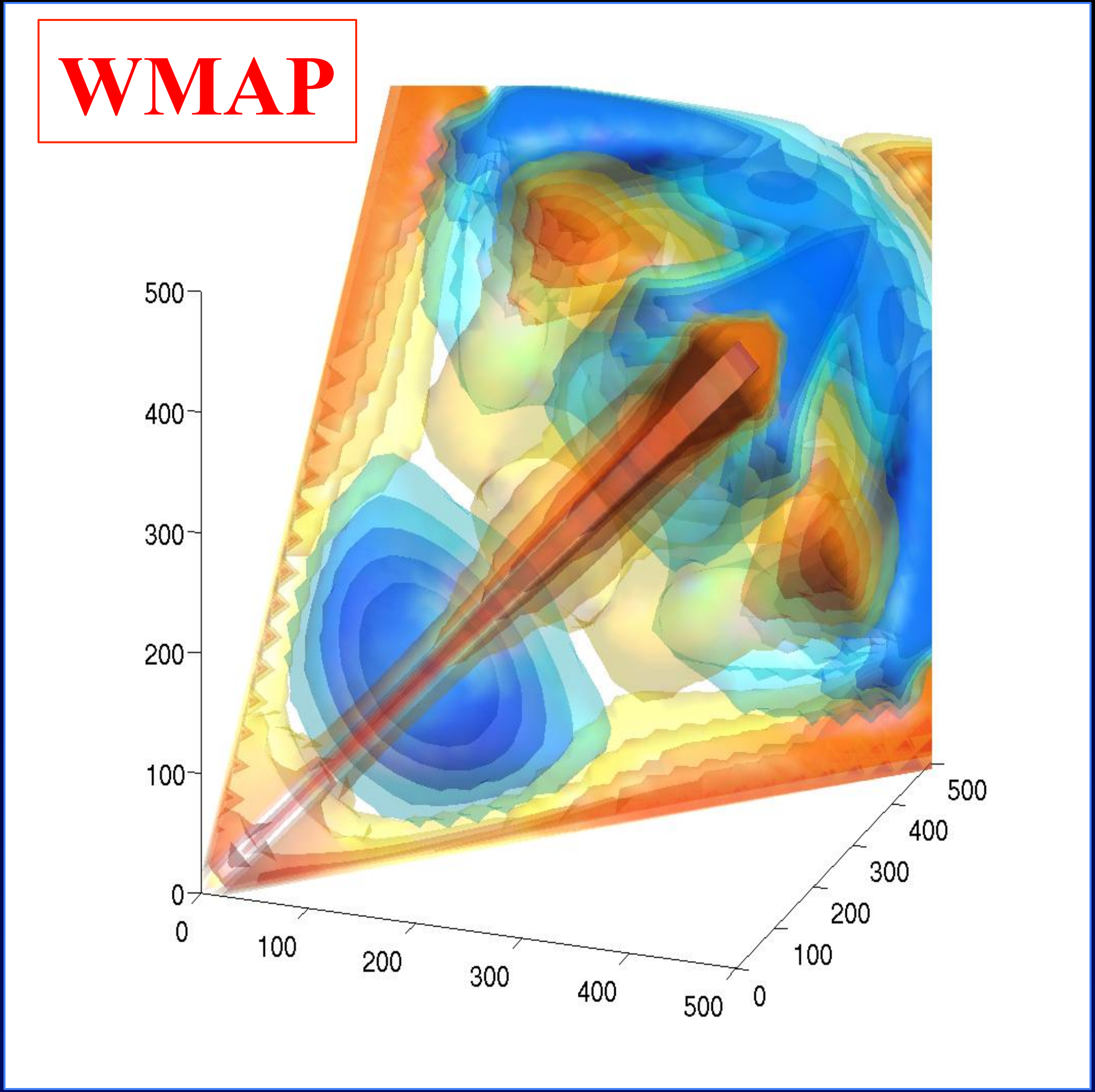
$$\hat{f}_{NL} = \frac{\vec{\alpha}_{shape} \cdot \vec{\beta}_{obs.}}{N}$$

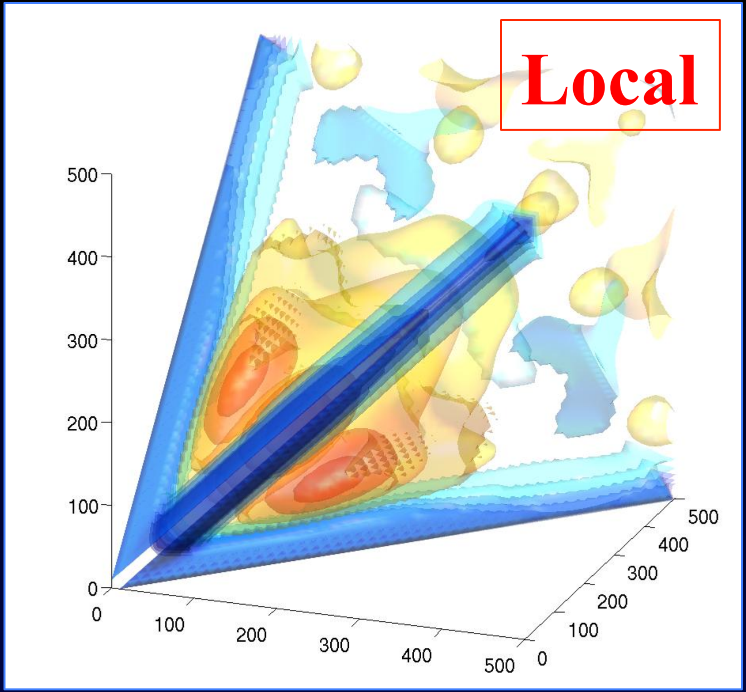
WMAP (5-year) mode reconstruction

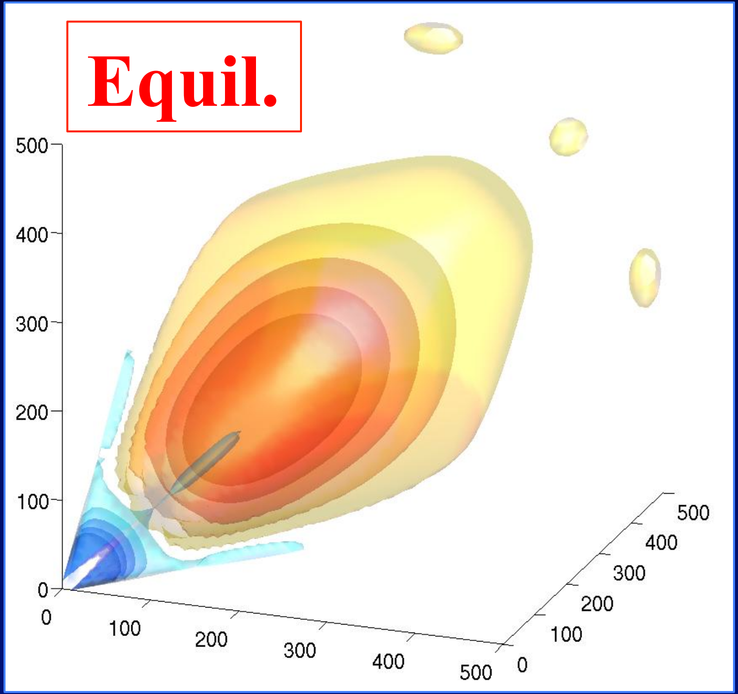
Fergusson, Liguori and Shellard 2010

- $l_{max} = 500$
- 31 modes
- V+W coadded map
- Inverse variance pre-filtering









WMAP shape reconstruction using first 31 modes (distance ordering). Comparison with local and equilateral

WMAP f_{NL} estimation

Scale invariant shapes:

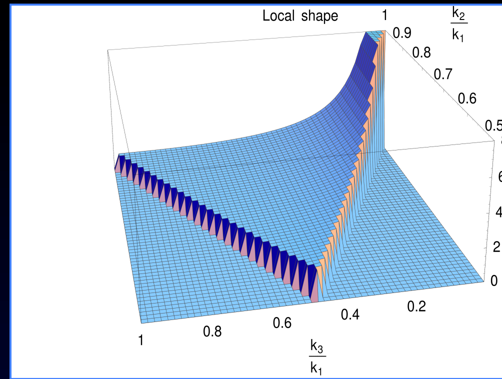
- ✓ Constant
- ✓ Local
- ✓ Warm
- ✓ Flat
- ✓ Equilateral family
  - Separable ansatz
  - DBI
  - Ghost inflation
  - Single field
- ✓ Orthogonal

Running shapes:

- ✓ Sharp feature in the inflaton potential

Local

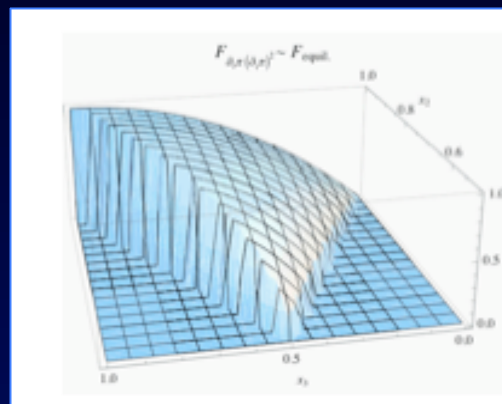
($f_{NL} = 39 \pm 20$)



$f_{NL} = 54.4 \pm 29.4$

Equilateral

($f_{NL} = 155 \pm 140$)



$f_{NL} = 143.5 \pm 151.2$ (sep.)

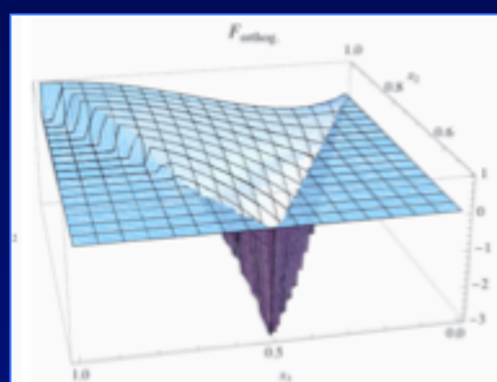
$f_{NL} = 146.0 \pm 144.5$ (DBI)

$f_{NL} = 138.7 \pm 165.4$ (ghost)

$f_{NL} = 142.1 \pm 131.2$ (single)

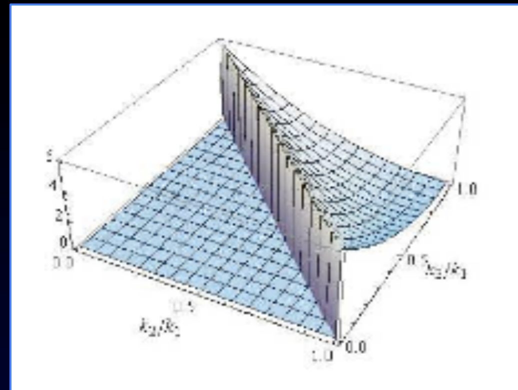
Orthogonal

($f_{NL} = -214 \pm 110$)



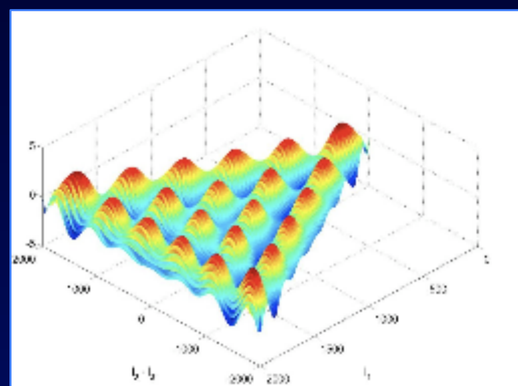
$f_{NL} = -79.4 \pm 133.3$

Flat



$f_{NL} = 18.1 \pm 14.9$

Constant



$f_{NL} = 149.4 \pm 116.8$

Scale invariance breaking feature

A step in the inflaton potential breaks slow-roll

⇩

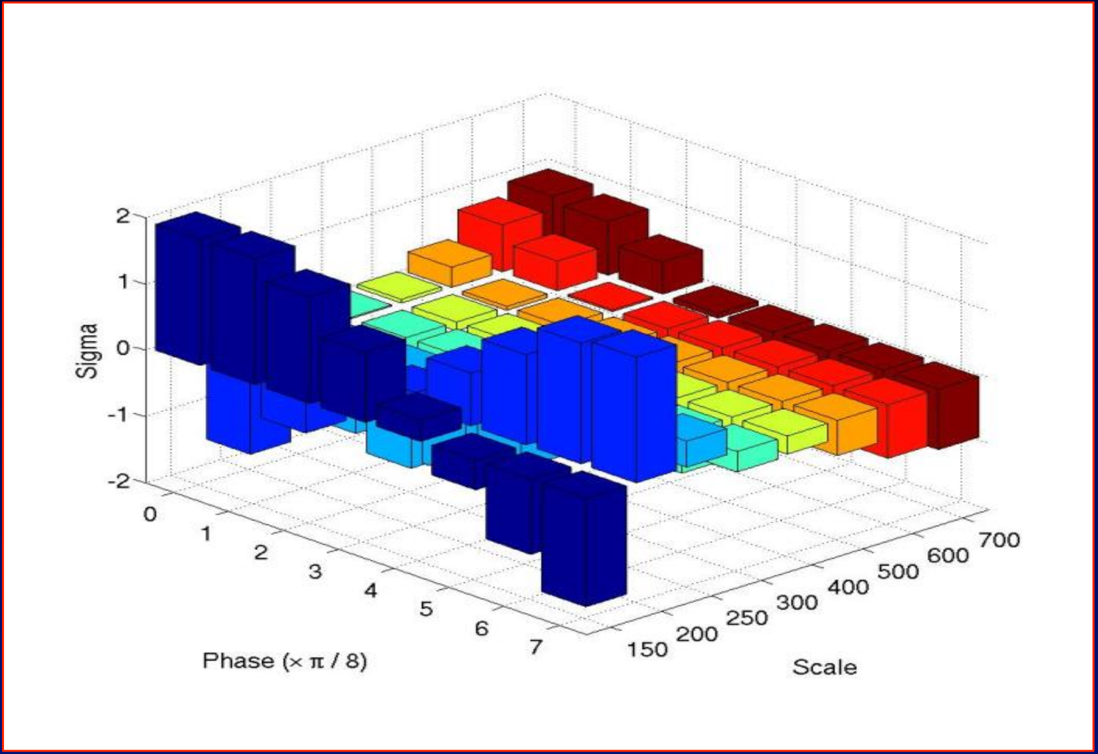
- ✓ Glitches in the power spectrum
- ✓ A large NG can be generated
- ✓ Scale invariance is broken

Sinusoidal running in the shape (parameters: amplitude, period, phase)

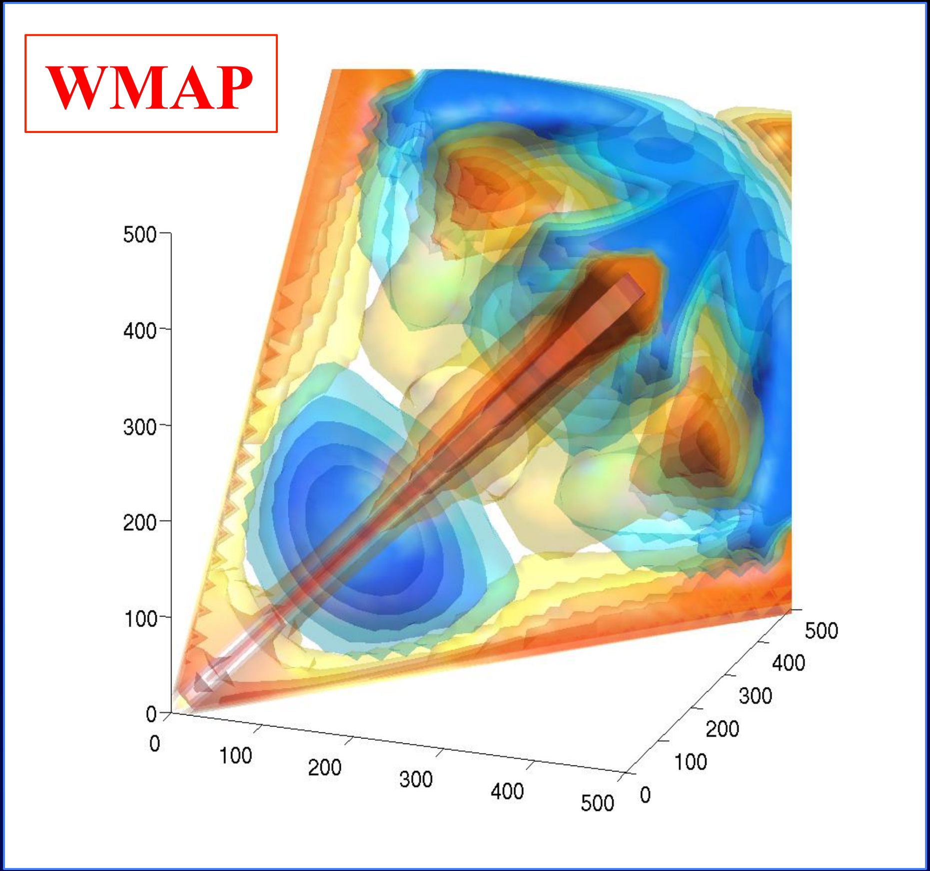
$$S \sim f_{NL}^{feat.} \sin\left(\frac{K}{k_*} + \phi\right)$$

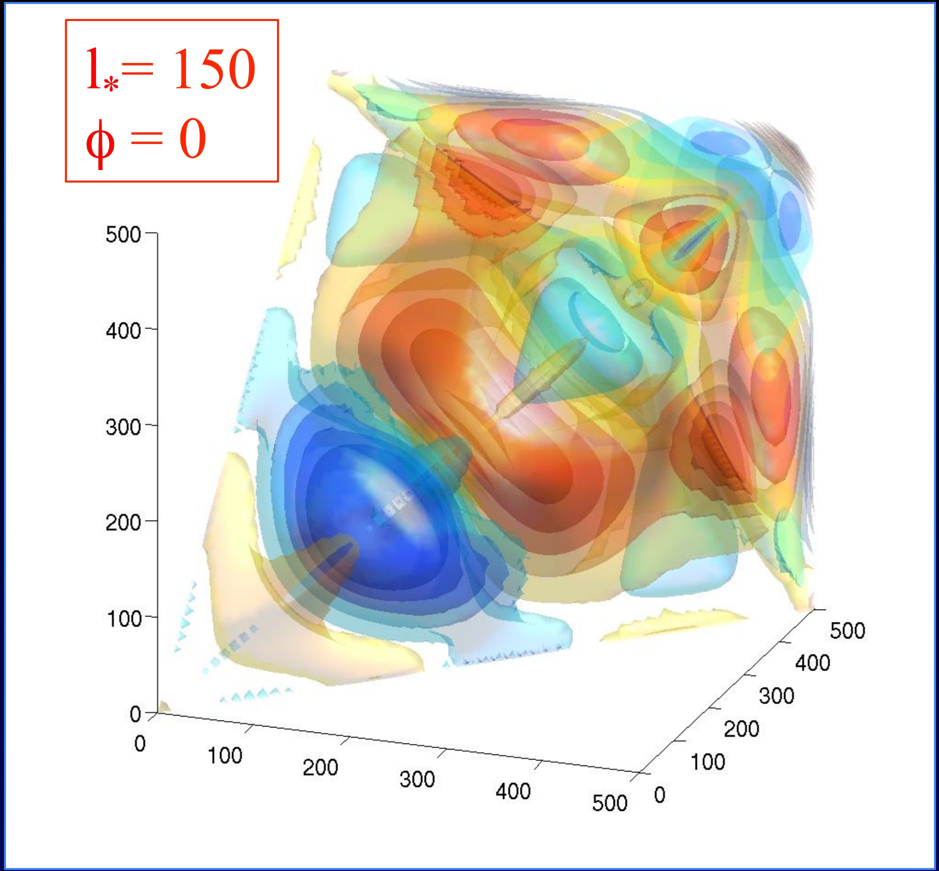
(Chen et al. 2006)

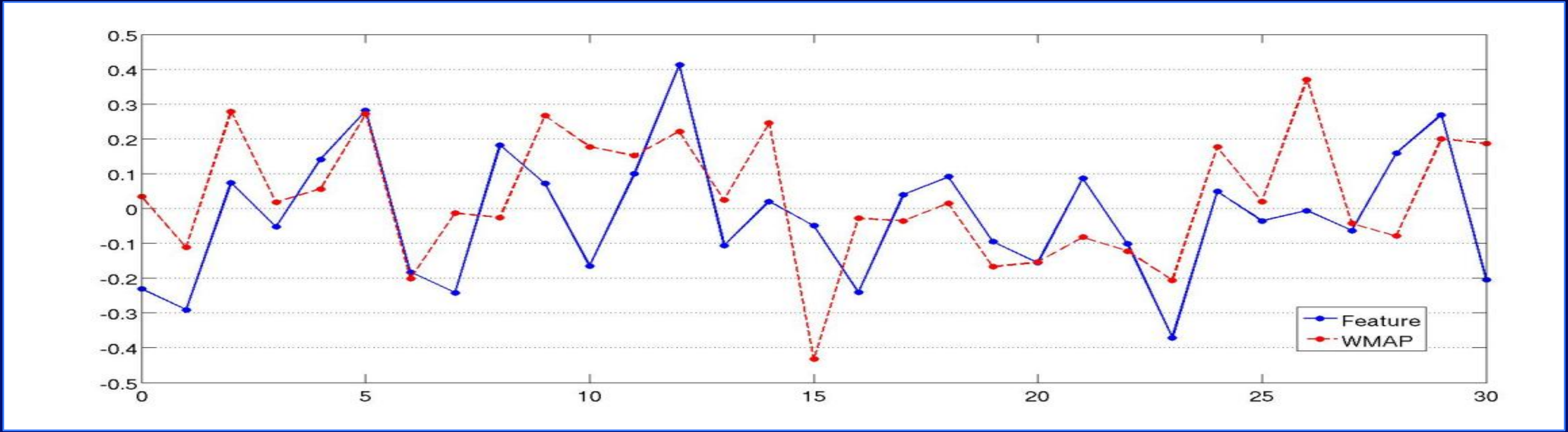
Phase \ Scale	150	200	250	300	400	500	600	700
0	57 (30)	−52 (33)	−25 (32)	1 (30)	1 (27)	8 (26)	18 (25)	23 (25)
$\pi/8$	67 (36)	−26 (27)	−36 (30)	−6 (25)	−4 (26)	−2 (27)	12 (26)	20 (25)
$\pi/4$	68 (42)	−10 (29)	−43 (30)	−11 (21)	−7 (25)	−10 (27)	−1 (28)	13 (27)
$3\pi/8$	49 (46)	7 (34)	−42 (32)	−18 (24)	−9 (25)	−14 (26)	−13 (28)	−2 (28)
$\pi/2$	15 (46)	32 (41)	−30 (35)	−32 (34)	−10 (25)	−16 (25)	−18 (27)	−14 (28)
$5\pi/8$	−19 (42)	63 (46)	−15 (35)	−38 (43)	−11 (25)	−16 (25)	−20 (26)	−20 (27)
$3\pi/4$	−39 (35)	87 (48)	0 (35)	−25 (41)	−11 (26)	−15 (25)	−21 (25)	−23 (26)
$7\pi/8$	−48 (30)	81 (43)	13 (34)	−11 (35)	−7 (27)	−13 (25)	−20 (25)	−23 (25)



- Amplitude computed for all phases and several scales between l=150 and l=700
- 64 combinations of scale and phase


WMAP
500
400
300
200
100
0
0
100
200
300
400
500
0
100
200
300
400
500


$l_* = 150$
$\phi = 0$
500
400
300
200
100
0
0
100
200
300
400
500
0
100
200
300
400
500


0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
5
10
15
20
25
30
Feature
WMAP

Mode decomposition at *Planck* resolution

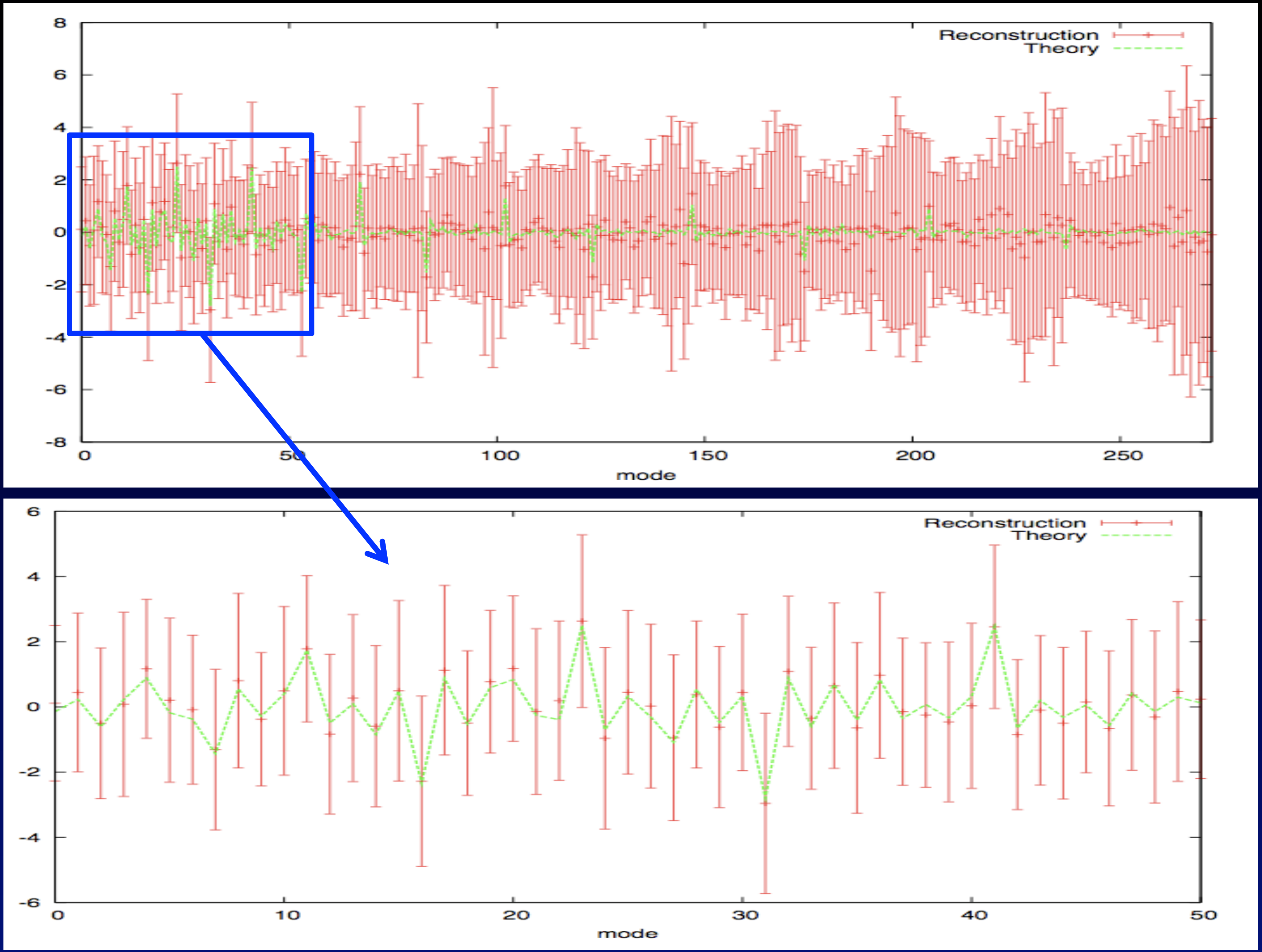
- Extend mode decomposition to much higher l_{max}. That requires many more modes.

l_{max}: 500 → 2000 n_{side} : 512 → 2048

p_{max}: 7 → 18 n_{modes} : 31 → 274

- Nothing conceptually different, but numerical stability and optimization issues required several technical modifications to the original WMAP pipeline.

- Currently being used on *Planck* data.


Reconstruction
Theory
mode
Reconstruction
Theory
mode

Summary

- ✓ We introduced an estimator of primordial NG based on a separable modal expansion of the bispectrum.

  Nice features of the modal estimator:

  1. It allows to separate any shape in a general clear mathematical framework.
  2. It allows *model independent* reconstruction of the 3-point function
  3. It makes multi-shape studies faster and simpler
  4. Through mode spectrum and shape reconstruction it allows a better monitoring of potential contaminants

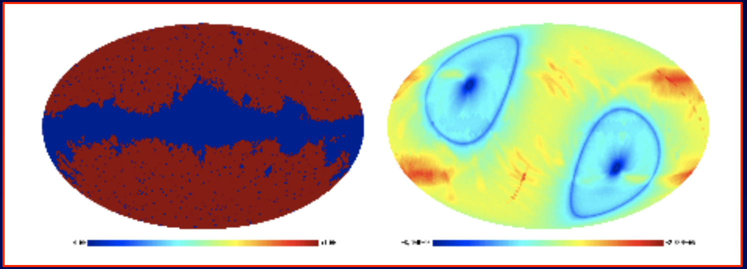
- ✓ We applied our estimators to WMAP 5-yr data and constrained a large number of models, *including first constraints on feature models*

- ✓ We extended our pipeline to high angular resolutions and we are now applying it to Planck data (as well as WMAP7)

Bispectrum estimator

A ML bispectrum estimator of f_{NL} has been shown to be optimal

$$\hat{f}_{NL} = \frac{1}{N} \sum_{\ell_i m_i} B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \frac{a^{m_1}_{\ell_1}}{C_{\ell_1}} \frac{a^{m_2}_{\ell_2}}{C_{\ell_2}} \frac{a^{m_3}_{\ell_3}}{C_{\ell_3}}$$



In presence of rotational invariance breaking terms

$$\hat{f}_{NL} = \frac{1}{N} \sum B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \left(C^{-1}a\right)^{m_1}_{\ell_1} \left(C^{-1}a\right)^{m_2}_{\ell_2} \left(C^{-1}a\right)^{m_3}_{\ell_3} - 3C^{-1}_{\ell_1 m_1 \ell_2 m_2} \left(C^{-1}a\right)^{m_3}_{\ell_3}$$

WMAP constraints

	WMAP 7-yrs	WMAP 5-yrs
Local	$-10 < f_{NL} < 74$	$-4 < f_{NL} < 80$
Equilateral	$-214 < f_{NL} < 266$	$-125 < f_{NL} < 435$
Orthogonal	$-410 < f_{NL} < 6$	$-369 < f_{NL} < 71$

(95% c.l)